INSTANTON SCREENING IN THE NONPERTURBATIVE GLUODYNAMICS

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Abstract

The gluon fields screening in the stochastic vacuum of gluodynamics is studied. The effective action is derived for the instanton interacting with nonperturbative fields. Quantum nonperturbative effects are shown to affect greatly the shape of instanton. The power asymptotics x^{-2} of the classical "instanton's profile function" at large distances is replaced due to these effects by Airy function asymptotics.

1 Introduction

In the last years a systematic description of nonperturbative effects in QCD has been provided in terms of the gluon nonlocal gauge–invariant correlators [1]. This Vacuum Correlators Method turned out to be very successful in phenomenological description of important QCD phenomena [2]. By now, there exist numerical results from lattice simulations concerning the fundamental field strength correlators for pure gauge theory with gauge group SU(2) [3] and SU(3) [4] over physical distances ranging up to O(1) fm.

On the other hand during the last 18 years we have witnessed active development of nonperturbative QCD picture based on the instanton liquid model [5,6,7]. It is therefore of great interest to compare the lattice calculations of field strength correlators with the computation of the same quantities in instanton–anti–instanton vacuum model. The analytical calculations of field strength correlators for the instanton–anti–instanton vacuum, at least, in the

dilute gas approximation exhibit the power decrease of the bilocal correlator at large distances. On the other hand, the results of lattice simulations are consistent with more rapidly decrease of this correlator. Thus we see that the computation of the field strength correlator requires the comprehensive analysis of instanton field character in nonperturbative vacuum. It is necessary to note that already in first publications on the instanton liquid model the Debye screening was revealed with a typical mass of the order of 350 MeV [6]. A new mechanism of instanton scale stabilization essentially different from that considered in [6] was proposed in [8,9]. It is due to the effective "freezing" of strong coupling constant in the nonperturbative vacuum.

In the present paper are obtained the effective action of topologically nontrivial fluctuations in the nonperturbative vacuum making use of the Vacuum Correlators Method. The new type is revealed of instanton screening different from previously considered [6,10].

2   Effective action

The Euclidean action of gluodynamics is written as

$$S = \frac{1}{2g^2} \int d^4x \, tr \mathcal{F}_{\mu\nu}^2(x), \tag{1}$$

where we use the Hermitian matrix form for gauge fields

$$\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu - i[\mathcal{A}_\mu, \mathcal{A}_\nu], \quad \mathcal{A}_\mu = g\mathcal{A}_\mu^a t^a, \quad tr t^a t^b = \frac{1}{2}\delta^{ab} \tag{2}$$

To obtain effective action for instanton in the nonperturbative vacuum we represent the full field $\mathcal{A}_\mu$ as

$$\mathcal{A}_\mu = A_\mu + B_\mu, \tag{3}$$

where A_μ is the external field (instanton–like object with unit topological charge) and B_μ are nonperturbative vacuum fields.

Under gauge transformations (U is the transformation matrix), the fields change as

$$A_\mu(x) \to U^+(x)A_\mu(x)U(x)$$
$$B_\mu \to U^+(x)(B_\mu(x) - i\partial_\mu)U(x) \tag{4}$$

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The general expression for the effective action of an instanton in the nonperturbative vacuum has the form

$$Z = e^{-S_{eff}[A]} = < e^{-S[A+B]+S[B]} >_B, \qquad (5)$$

where

$$< \hat{0}(B) >_B = \int d\mu[B]\hat{0}(B) \qquad (6)$$

and $d\mu[B]$ is the measure of integration with respect to nonperturbative fields. The explicit form of this measure will not be needed in our analysis. The full field strength can be rewritten in the form

$$\mathcal{F}_{\mu\nu}[A + B] = F_{\mu\nu}[A] + G_{\mu\nu}[B] - i[A_\mu, B_\nu] - i[B_\mu, A_\nu] \qquad (7)$$

Let us now consider a large–size instanton. The squared strength of the instanton field at the center is given by

$$(F_{\mu\nu}^{inst}(x = x_0))^2 = 192/\rho^4 \qquad (8)$$

For $\rho > \rho_0 = (192/ < G^2 >)^{1/4} \sim 1fm$, the instanton field is weak relative to the characteristic field strengths in the vacuum gluon condensate $< G^2 > \equiv < (gG_{\mu\nu}^a)^2 > \simeq 0.5 Gev^4$ [11] and can be treated as a perturbation. Therefore, expanding the effective action (5) up to the second–order terms in A_μ we have

$$S_{eff}[A] = -lnZ \simeq S[A] + < S_{dia}[A, B] >_B + < S_{para}[A, B] >_B, \qquad (9)$$

where

$$S_{dia}[A, B] = -\frac{1}{2g^2} \int d^4x tr\{[A_\mu, B_\nu] + [B_\mu, A_\nu]\}^2, \qquad (10)$$

$$S_{para}[A, B] = -\frac{1}{4g^2} \int d^4x d^4y tr F_{\mu\nu}(x) G_{\mu\nu}(x) tr F_{\sigma\lambda}(y) G_{\sigma\lambda}(y), \qquad (11)$$

and the conditions $< B_\mu >= 0, < G_{\mu\nu} >= 0$ are taken into account.

Hence, there exist two contributions [9]. First, there is the motion of "charged particles" in the external gluonic field $F_{\mu\nu}$. This motion leads to screening similar to orbital Landau diamagnetism. And second, there is the direct interaction of $F_{\mu\nu}$ with the spin of the field B_μ. In the second order in the external field, this interaction yields an antiscreening term, which is analogous to that describing the Pauli paramagnetic effect.

A similar decomposition into diamagnetic and paramagnetic terms was suggested by Polyakov [12] for perturbative gluodynamics, and led to the simple and elegant explanation of antiscreening with correct coefficient for β–function.

Let us consider paramagnetic component of S_{eff}. The general form of the instanton field is

$$A_\mu(x) = \Phi(x, x_0)\Omega\bar{A}_\mu(x - x_0))\Omega^+\Phi(x_0, x), \tag{12}$$

where

$$\Phi(x, x_0) = P \ exp\{i \int_{x_0}^x B_\mu dz_\mu\} \tag{13}$$

is the operator of parallel transport, Ω is the matrix of global rotations in the color space and $\bar{A}_\mu(x - x_0)$ in the equation (12)

$$\bar{A}_\mu = \tau^a \bar{\eta}^a_{\mu\nu} \frac{x_\nu}{x^2} f(x^2) \tag{14}$$

is the instanton field in some gauge. The f is so-called "instanton's profile function". Classical function f has the standard form in the singular gauge,

$$f_{cl} = \frac{1}{1 + (x^2/\rho^2)} \tag{15}$$

Prior to averaging over the fields B_μ, we write down the paramagnetic term in S_{eff} in the following form[1]

$$S_{para}[A, B] = -\frac{1}{2g^4} \int d^4x d^4y \int d\Omega tr\Omega F_{\mu\nu}(x)\Omega^+ G_{\mu\nu}(x, x_0)$$

$$\times tr\Omega F_{\sigma\lambda}(y)\Omega^+ G_{\sigma\lambda}(y, x_0), \tag{16}$$

where

$$G_{\mu\nu}(x, x_0) \equiv \Phi(x_0, x)G_{\mu\nu}(x)\Phi(x, x_0) \tag{17}$$

and $d\Omega$ is the Haar measure on the $SU(N_c)$. In the tensor notation, the integrand in (16) is written as

$$\Omega_{\alpha_1\beta_1}(F_{\mu\nu})_{\beta_1\alpha_2}\Omega^+_{\alpha_2\beta_2}(G_{\mu\nu})_{\beta_2\alpha_1}\Omega_{\alpha_3\beta_3}(F_{\sigma\lambda})_{\beta_3\alpha_4}\Omega^+_{\alpha_4\beta_4}(G_{\sigma\lambda})_{\beta_4\alpha_3} \tag{18}$$

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[1]In this section the notation $\bar{A}_\mu = A_\mu$ for brevity is used.

Taking into account the relation

$$\int d\Omega \, \Omega_{\alpha_1\beta_1} \Omega^+_{\alpha_2\beta_2} \Omega_{\alpha_3\beta_3} \Omega^+_{\alpha_4\beta_4}$$

$$= \frac{1}{N_c^2 - 1} \{ \delta_{\alpha_1\beta_2} \delta_{\beta_1\alpha_2} \delta_{\alpha_3\beta_4} \delta_{\beta_3\alpha_4} + \delta_{\alpha_1\beta_4} \delta_{\beta_1\alpha_4} \delta_{\alpha_2\beta_3} \delta_{\beta_2\alpha_3} \} + O(\frac{1}{N_c^3}) \qquad (19)$$

and integrating (16) over the Haar measure, one obtains

$$S_{para}[A, B] = -\frac{1}{2(N_c^2 - 1)g^4} \int d^4x d^4y \, tr F_{\mu\nu}(x) F_{\sigma\lambda}(y) tr G_{\mu\nu}(x, x_0) G_{\sigma\lambda}(y, x_0). \tag{20}$$

Let us perform averaging over the nonperturbative field B_μ and remind that the nonlocal gauge–invariant correlation function $< tr G_{\mu\nu}(x, x_0) G_{\sigma\lambda}(y, x_0) >$ can be represented as [1]

$$< tr G_{\mu\nu}(x, x_0) G_{\sigma\lambda}(y, x_0) >= \frac{< G^2 >}{12} \{ (\delta_{\mu\sigma}\delta_{\nu\lambda} - \delta_{\mu\lambda}\delta_{\nu\sigma}) D(x - y) + O(D_1) \}. \tag{21}$$

In this way, the paramagnetic term in S_{eff} is reduced to the form

$$S_{para}[A] =< S_{para}[A, B] >_B = -\frac{< G^2 >}{48(N_c^2 - 1)g^4} \int d^4x d^4y \, F^a_{\mu\nu}(x) D(x-y) F^a_{\mu\nu}(y). \tag{22}$$

Let us consider the diamagnetic part

$$S_{dia}[A, B] = -\frac{1}{g^2} \int d^4x \, tr\{ [A_\mu, B_\nu]^2 + [A_\mu, B_\nu][B_\mu, A_\nu] \}. \tag{23}$$

Again we integrate over the Haar measure first. Thus, the S_{dia} is given by the following expression:

$$S_{dia}[A, B] = \frac{3}{2g^2} \frac{N_c}{N_c^2 - 1} \int d^4x \, tr A_\mu^2(x) tr B_\mu^2(x). \tag{24}$$

To obtain gauge–invariant expressions for vacuum correlator, we use the Fock–Schwinger gauge $x_\mu B_\mu = 0$. By virtue of the fact that the t'Hooft symbols $\bar\eta^a_{\mu\nu}$ are antisymmetric, the instanton field satisfies this gauge condition automatically. In this gauge, the nonperturbative field B_μ is related to the field strength $G_{\mu\nu}$ by the well–known equation

$$B_\mu(x) = x_\nu \int_0^1 \alpha d\alpha G_{\mu\nu}(\alpha x). \tag{25}$$

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Using the relation (21) we arrive at

$$S_{dia}[A] = <S_{dia}[A, B]>_B = \frac{1}{2g^2} \int d^4x \Sigma(x)(A_\mu^a(x))^2, \qquad (26)$$

where

$$\Sigma(x) = \frac{3}{16} \frac{N_c}{N_c^2 - 1} <G^2> x^2 \int_0^1 \alpha d\alpha \int_0^1 \beta d\beta D(|\alpha - \beta|x). \qquad (27)$$

Bringing all the results together, we find that S_{eff} for instanton–like object in the nonperturbative vacuum is given by

$$S_{eff}[A] = S[A] + S_{para}[A] + S_{dia}[A]$$

$$= \frac{1}{4g^2} \int d^4x d^4y F_{\mu\nu}^a(x)\varepsilon(x-y)F_{\mu\nu}^a(y) + \frac{1}{2g^2} \int d^4x \Sigma(x)(A_\mu^a(x))^2, \qquad (28)$$

where

$$\varepsilon(x-y) = \delta^4(x-y) - \Pi(x-y), \quad \Pi(x-y) = \frac{<G^2>}{12(N_c^2 - 1)g^2} D(x-y) \quad (29)$$

Equation of motion for A_μ is

$$\nabla_\mu^{ab}(A(x)) \int d^4y \varepsilon(x-y)F_{\mu\nu}^b(y) - \Sigma(x)A_\nu^a(x) = 0 \qquad (30)$$

and

$$\nabla_\mu^{ab} = \delta^{ab}\partial_\mu - if^{abc}A_\mu^c$$

is the covariant derivative on the gauge group $SU(N_c)$.

3   Scale transformations

In the previous section we have derived S_{eff} for weak external field (instanton–like object of large size) in the nonperturbative gluon vacuum. It is known that scale invariance in gluodynamics is broken due to the anomaly of the trace of energy–momentum tensor $<\theta_{\mu\mu}> \sim - <G^2>$. Topology allows the existence of topologically nontrivial solutions. However, their stability in size is not evident from scale arguments. For example, in theories which include Higgs fields (Weinberg–Salam or Georgi–Glashow models) instanton

solutions are absent at the classical level. Instanton tends to contract in size thus lowering the classical action and finally becoming a point singularity with $\rho = 0$.

If however one considers the effective action S_{eff} with quantum corrections over the instanton background field, one can perform a scale transformation $x_\mu \to x_\mu/\lambda, \quad A_\mu(x) \to \lambda^{-1} A_\mu(x/\lambda)$ and then consider the derivative of S_{eff} with respect of the logarithm of the transformation parameter. This yields

$$\left.\frac{\partial S_{eff}}{\partial ln\lambda}\right|_{\lambda=1} = \int d^4 x \theta_{\mu\mu}(x) \quad , \tag{31}$$

where $\theta_{\mu\mu}(x)$ is the trace of energy–momentum tensor for a given field configuration. The trace $\theta_{\mu\mu}(x)$ contains a positively defined contribution which is due to the mass term of the classical action. This mass term breaks scale invariance and leads to the collapse of the classical instanton. In the quantum theory $\theta_{\mu\mu}$ acquires a negative contribution due to quantum anomaly. It is the presence of this anomaly which eventually stabilizes the instanton. One may argue that the size of the instanton is determined from the condition of the total pressure to be zero

$$\int d^4 x \theta_{\mu\mu}(x) = 0.$$

Let us consider the scale transformation of the effective action (28) and prove the existence of the stable in size instanton in nonperturbative scale noninvariant gluodynamics.

Numerical lattice calculations revealed that, to a high accuracy, the function D can be approximated as [3,4]

$$D(x - y) = e^{-\mu|x-y|}, \quad \mu \equiv 1/T_g \approx 1 GeV. \tag{32}$$

Let write (22) in the Fourier form

$$S_{para}[A] = -\frac{1}{4g^2} \int \frac{d^4 q}{(2\pi)^4} F_{\mu\nu}^a(q) F_{\mu\nu}^a(-q) \Pi(q^2). \tag{33}$$

Taking into account (29) one obtains

$$\Pi(q^2) = \frac{< G^2 >}{12(N_c^2 - 1)g^2} \int d^4 x e^{iqx} D(x) = \frac{< G^2 >}{(N_c^2 - 1)g^2} \frac{\pi^2 \mu}{(\mu^2 + q^2)^{5/2}}. \tag{34}$$

Perform now a scale transformation $q_\mu \to \lambda q_\mu$, $F_{\mu\nu}^a(q) \to \lambda^2 F_{\mu\nu}^a(\lambda q)$. Then

$$\frac{\partial S_{para}}{\partial ln\lambda}\bigg|_{\lambda=1} = \frac{1}{2g^2} \int \frac{d^4q}{(2\pi)^4} F_{\mu\nu}^a(q) F_{\mu\nu}^a(-q) \frac{\partial \Pi(q^2)}{\partial lnq^2} < 0. \tag{35}$$

Consider S_{dia} in the same way. Performing a transformation $x_\mu \to x_\mu/\lambda$, $A_\mu(x) \to \lambda^{-1} A_\mu(x/\lambda)$ one arrives at

$$\frac{\partial S_{dia}}{\partial ln\lambda}\bigg|_{\lambda=1} = \frac{1}{g^2} \int d^4x (A_\mu^a(x))^2 (\Sigma(x) + \frac{\partial \Sigma(x)}{\partial lnx^2}). \tag{36}$$

Making use of the relations (27) and (32) and of the conclusions on $D(|\alpha - \beta|x)$ from [9] one can deduce that $\Sigma(x) + \partial\Sigma(x)/\partial lnx^2 > 0$. Therefore

$$\frac{\partial S_{dia}}{\partial ln\lambda}\bigg|_{\lambda=1} > 0. \tag{37}$$

It means that diamagnetic interaction of the instanton with nonperturbative fields leads to the collapse of the instanton. On the other hand the paramagnetic interaction effectively results in instanton "swelling" in its scale. Hence a stable in size instanton can exist in the nonperturbative gluonic vacuum described by nonlocal gauge–invariant correlator $< trG_{\mu\nu}(x, x_0) G_{\sigma\lambda}(y, x_0) >$.

4   Screening

The instanton profile function in the nonperturbative vacuum is determined by Eq. (30) for the ansatz (14). At small distances from the center of the instanton $x \to 0, (q \to \infty)$ equations (27), (29) and (34) lead to [2]

$$\Sigma(x \to 0) \to \frac{1}{16} \frac{N_c}{N_c^2 - 1} < G^2 > x^2,$$

$$\varepsilon(q \to \infty) \to 1 \tag{38}$$

So at small distances one retrieves the classical equation for the profile function

$$\frac{d^2f}{dx^2} + 2\frac{df}{dx} - \frac{8}{x}(2f^3 - 3f^2 + f) = 0, \quad f(0) = 1, f(\infty) = 0 \tag{39}$$

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[2]In this section, we use the notation $x = \sqrt{x_\mu^2}$.

and, correspondingly, $f(x \to 0) \to f_{cl}(x)$ (15).

At large distances $x \to \infty, (q \to 0)$

$$\Sigma(x \to \infty) \simeq \frac{1}{8} \frac{N_c}{N_c^2 - 1} < G^2 > T_g x (1 + O(\frac{T_g}{x}))$$

$$\varepsilon(q \to 0) \simeq 1 - \frac{\pi^2}{(N_c^2 - 1)g^2} < G^2 > T_g^4 \qquad (40)$$

Thus, we have the following asymptotic equation for the profile function

$$\frac{d^2 f}{dx^2} - \gamma x f = 0, \quad f(\infty) = 0, \qquad (41)$$

where

$$\gamma = \frac{1}{8} \frac{N_c}{N_c^2 - 1} < G^2 > T_g (1 - \frac{\pi^2}{(N_c^2 - 1)g^2} < G^2 > T_g^4)^{-1} \qquad (42)$$

Hence the function f is asymptotically damped at large distances like the Airy function

$$f(x) \sim Ai(\gamma^{1/3} x) \to \frac{1}{2} \sqrt{\frac{1}{\pi}} \gamma^{-1/12} x^{-1/4} exp(-\frac{2}{3} \gamma^{1/2} x^{3/2}), \quad x \to \infty \qquad (43)$$

One notice that from (43) we have screening of the instanton profile function with effective mass $m_{sc} = (2/3)^{2/3} \gamma^{1/3} \approx \gamma^{1/3}$. Choosing the standard values of the parameters in QCD $N_c = 3, T_g = 1 GeV^{-1}, < G^2 >= 0, 5 GeV^4, \alpha_s = g^2/4\pi = 0, 3$ one obtains the screening mass $m_{sc}(QCD) \simeq 250 MeV$. On the other hand, as it is well–known, in gluodynamics the value of the vacuum condensate is $< G^2 >_{GD} \simeq 4 < G^2 >$ [13]. Therefore, one obtains the screening mass $m_{sc}(GD) \simeq 520 MeV$.

5 Conclusion

Thus, making use of Vacuum Correlators Method we have obtained the effective action for the topologically nontrivial gluon field fluctuation (instanton–like object) interacting with stochastic vacuum fields. Effective action appears to be essentially nonlocal. This nonlocal interaction (paramagnetic effect) leads eventually to the existence of the instanton which is scale non-collapsing.

It should be stressed that we have used exponentially decreasing ansatz (32) for the D–function. However, it can be easily seen that our result is valid for any function D that decrease sufficiently fast. The only difference is in the numerical value of γ. For example, for Gaussian ansatz one has $D(z) = \exp(-\mu^2 z^2)$, $\gamma \to C \cdot \gamma$ and $C \approx 0.9$.

One concludes that the effect of screening is able to change drastically the interaction in the instanton liquid (dipole–dipole attraction at the large distances), where the average distances between pseudoparticles $(\sim (200 MeV)^{-1})$ are larger than the inverse screening mass $(\sim (500 MeV)^{-1})$. The numerical calculations for the instanton screening effects in the nonperturbative gluodynamics vacuum and comparison with the lattice calculations are in progress.

Acknowledgments

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